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# A class of irreversible Carnot refrigeration cycles with a general heat transfer law

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**Abstract.** The performance of a class of irreversible Carnot refrigeration cycles operating between two heat reservoirs at a low temperature  $T_L$  and a high temperature  $T_H$ , for which the only irreversibility results from the finite rate of heat conduction, is studied. The relation between the optimal rate of refrigeration and the coefficient of performance of these cycles is derived, based on a general heat transfer law. Moreover, the optimal performance of these cycles is discussed and the effect of different heat transfer laws on the optimal performance is investigated. Consequently, some new and useful results for refrigeration cycles are obtained.

## 1. Introduction

It has been demonstrated by many authors that under different heat transfer laws irreversible cycles have different optimal configurations and characters [1–5]. For example, for a Carnot cycle operating between two heat reservoirs at a high temperature  $T_H$  and a low temperature  $T_L$ , and with the heat conductances  $K_1$  and  $K_2$  between the working material and the two heat reservoirs, when heat transfer obeys Newton's law (i.e. the heat flux  $q \propto \Delta T$ ), the efficiency of the cycle at maximum power output is

$$\eta_m = 1 - (T_L/T_H)^{1/2}. \quad (1)$$

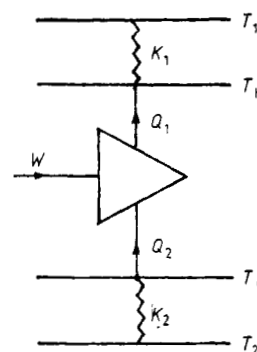
This is the CA efficiency [6], just as the Carnot efficiency is only a function of reservoir temperatures. When heat transfer obeys another linear heat transfer law in irreversible thermodynamics (i.e.  $q \propto \Delta(1/T)$ ), the efficiency of the cycle at maximum power output as shown previously [5] is

$$\eta_m = (\sqrt{K_1} + \sqrt{K_2})(1 - T_L/T_H)/(\sqrt{K_1} + 2\sqrt{K_2} + \sqrt{K_1}T_L/T_H). \quad (2)$$

It is not only a function of reservoir temperatures, but also of the heat conductances. As another example, when the heat transfer law is

$$q = \alpha(1/T - 1/T_R) + \beta(1/T - 1/T_R)^9 \quad (3)$$

(where  $\alpha > 0$  and  $\beta > 0$  are the heat conductances, and  $T$  and  $T_R$  are the temperatures of the system and



**Figure 1.** A Carnot refrigeration cycle affected by thermal resistance.

reservoirs, respectively) the optimal cycle of maximum efficiency with a given amount of heat input is no longer Carnot's form, but consists of three isotherms and three adiabats [1].

For refrigeration cycles, there is naturally the same problem as power cycles. However, up to now the effects of heat transfer law on the performance of refrigeration cycles have never been investigated systematically. Therefore, it is very worthwhile to study the problem further.

In this paper the relation between the optimal rate of refrigeration and the coefficient of performance of a class of Carnot refrigeration cycles, whose only irreversibility results from heat conduction, is derived based on a general heat transfer law. Moreover, the optimal performances of such a class of cycles for three

common heat transfer laws are discussed. In particular, the optimal performance of a Newton's law Carnot refrigeration cycle is analysed in detail. It is pointed out that the effect of thermal resistance is a serious problem in ultra-low-temperature technology and brings about twofold difficulties in achieving ultra-low temperatures. At the same time, the inherent and intrinsic character of the effect of the heat transfer law on the performance of such a class of cycles is also revealed.

## 2. The model of a class of irreversible Carnot refrigeration cycles

The model of the type of irreversible Carnot refrigeration cycles considered in this paper is shown in figure 1, and satisfies the following conditions.

(i) In such a model, a reversible Carnot refrigeration cycle is carried out inside the working material.

(ii) The only irreversibility in the model results from the finite rate of heat conduction between the working material and the two heat reservoirs. When the two isothermal processes inside the working material are carried out, their temperatures  $T_1$  and  $T_2$  are different from the reservoir temperatures  $T_H$  and  $T_L$ , and there exists the relation  $T_1 > T_H > T_L > T_2$ . Thus, the heat conduction may be carried out in finite time, and the cycle may have a certain rate of refrigeration.

(iii) The isentropic processes occur in negligible time. This means that they must occur on a timescale that is fast compared with the slow rates for heat leaks to the environment, but slow compared with the rapid internal relaxation of pressure gradients in the working material [7]. If this is the case, cycle time is given by

$$\tau = t_1 + t_2 \quad (4)$$

where  $t_1$  and  $t_2$  are the times of releasing and absorbing heat processes respectively.

This model is similar to the Curzon–Ahlborn cycle model which has been extensively adopted by many authors [8–10], but the model adopted here is a refrigeration cycle and is somewhat different. The temperatures  $T_1$  and  $T_2$  of the two isothermal processes inside the working material are not between  $T_H$  and  $T_L$ . They satisfy the relation  $T_1 > T_H > T_L > T_2$ . Therefore, for a given  $T_H$  and  $T_L$ , the path of such refrigeration cycles cannot coincide with that of a Curzon–Ahlborn cycle. This is different from a reversible Carnot cycle in which the path of the reverse cycle is coincident with the original one. This just indicates that the Carnot refrigeration cycles mentioned above are irreversible.

In order to make a unified description of the various optimum performances of the above Carnot refrigeration cycles under several common heat transfer laws, we use a general heat transfer law which has been adopted by Vos [2, 4] and Chen [5], i.e., the heat  $Q_1$

released to the high-temperature reservoir and the heat  $Q_2$  absorbed from the low-temperature reservoir by the working material per cycle are assumed to satisfy the following relations:

$$Q_1 = K_1(T_1^n - T_H^n)t_1 \quad (5)$$

and

$$Q_2 = K_2(T_L^n - T_2^n)t_2. \quad (6)$$

$K_1$  and  $K_2$  are the heat conductances between the working material and the two heat reservoirs at temperatures  $T_H$  and  $T_L$ .  $n$  is a non-zero integer. When  $n < 0$ ,  $K_1$  and  $K_2$  are negative. The generality and significance of such a heat transfer law lies in the fact that, when a different value of  $n$  is chosen, it represents a different heat transfer law. In particular,  $n = 1$  represents Newton's law,  $n = -1$  stands for another linear heat transfer law in irreversible thermodynamics and  $n = 4$  stands for the thermal radiation law. They are often used in practice and for this reason, we take equations (5) and (6) as general heat transfer laws.

According to the model in which a reversible Carnot refrigeration cycle is carried out inside the working material, one has

$$Q_2/Q_1 = T_2/T_1 \quad (7)$$

and the coefficient of performance is

$$\varepsilon = Q_2/(Q_1 - Q_2) = T_2/(T_1 - T_2). \quad (8)$$

Using equations (4)–(7), one can obtain the average rate of refrigeration of the cycle as

$$\begin{aligned} R = Q_2/\tau &= K_2(T_L^n - T_2^n)/(1 + t_1/t_2) \\ &= \frac{K_2}{\frac{1}{T_L^n - T_2^n} + \frac{K_2 T_1}{K_1 T_2} \left( \frac{1}{T_1^n - T_H^n} \right)}. \end{aligned} \quad (9)$$

## 3. The relation between optimal rate of refrigeration and coefficient of performance

In order to find the relation between the optimal rate of refrigeration and the coefficient of performance, equation (8) is written as

$$T_1/T_2 = 1 + \varepsilon^{-1}, \quad (10)$$

and substitution of equation (10) into equation (9) gives

$$R = \frac{K_2}{\frac{1}{T_L^n - T_2^n} + \frac{K_2}{K_1} \frac{1 + \varepsilon^{-1}}{(1 + \varepsilon^{-1})^n T_2^n - T_H^n}}. \quad (11)$$

Using the extremal condition

$$(\partial R / \partial T_2)_\varepsilon = 0 \quad (12)$$

we can find

$$\begin{aligned} (T_L^n - T_2^n)^{-2} &= (K_2/K_1)(1 + \varepsilon^{-1})^{n+1} \\ &\times [(1 + \varepsilon^{-1})^n T_2^n - T_H^n]^{-2} \end{aligned} \quad (13)$$

from equation (11). Solving equation (13), we get

$$T_2^n = \frac{T_H^n + [(K_2/K_1)(1 + \varepsilon^{-1})^{n+1}]^{1/2} T_L^n}{(1 + \varepsilon^{-1})^n + [(K_2/K_1)(1 + \varepsilon^{-1})^{n+1}]^{1/2}}. \quad (14)$$

Then, we obtain

$$T_L^n - T_2^n = \frac{T_L^n - T_H^n/(1 + \varepsilon^{-1})^n}{1 + [K_2/K_1(1 + \varepsilon^{-1})^{n-1}]^{1/2}} \quad (15)$$

and

$$(1 + \varepsilon^{-1})^n T_2^n - T_H^n = \frac{[K_2(1 + \varepsilon^{-1})^{n+1}/K_1]^{1/2} [T_L^n - T_H^n/(1 + \varepsilon^{-1})^n]}{1 + [K_2/K_1(1 + \varepsilon^{-1})^{n-1}]^{1/2}}. \quad (16)$$

Finally, substitution of equations (15) and (16) into equation (11) gives the relation between the optimal rate of refrigeration  $R$  and the coefficient of performance  $\varepsilon$ :

$$R = \frac{K_2[T_L^n - T_H^n/(1 + \varepsilon^{-1})^n]}{\{1 + [K_2/K_1(1 + \varepsilon^{-1})^{n-1}]^{1/2}\}^2}. \quad (17)$$

Since

$$(\partial \varepsilon / \partial T_2)_R = -(\partial R / \partial T_2)_\varepsilon / (\partial R / \partial \varepsilon)_{T_2} \quad (18)$$

the condition  $(\partial \varepsilon / \partial T_2)_R = 0$  corresponds to equation (12) under the circumstances  $(\partial R / \partial \varepsilon)_{T_2} \neq 0$ . Therefore, equation (17) also determines the relation between the optimal coefficient of performance and the rate of refrigeration. In fact, equation (17) is a general and fundamental relation for the model cycles. The various optimum performances of such a model cycle and the effect of the heat transfer law on these performances can all be deduced from it.

#### 4. Optimal performance of the model cycles for three common heat-transfer laws

##### 4.1. Case $n = 1$

The case  $n = 1$  is an important heat transfer model which is extensively used in practice and finite-time thermodynamics [6–14]. When  $n = 1$ , it can be obtained, from equation (17), that the relation between the optimal rate of refrigeration and the coefficient of performance, or the relation between the optimal coefficient of performance and the rate of refrigeration, of the model cycles is

$$R = K[T_L - \varepsilon T_H / (1 + \varepsilon)] \quad (19)$$

or

$$\varepsilon = (T_L - R/K) / (T_H - T_L + R/K) \quad (20)$$

where  $K = K_1 K_2 / (\sqrt{K_1} + \sqrt{K_2})^2$ .

From equation (19) or (20), we can find the following important conclusions.

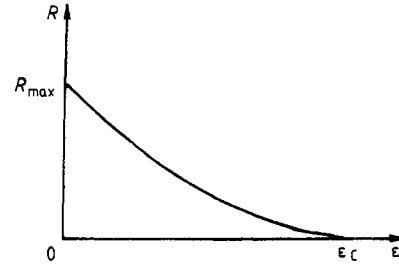


Figure 2. The curve  $R$  varying with  $\varepsilon$  for  $n = 1$ .

(i) The relation between  $R$  and  $\varepsilon$  determined by equation (19) or (20) is shown in figure 2. Figure 2 indicates clearly that  $R$  (or  $\varepsilon$ ) decreases monotonically as  $\varepsilon$  (or  $R$ ) increases. Consequently, consideration must be given to both the coefficient of performance and the rate of refrigeration of a refrigerator.

(ii) Only if  $R = 0$  can  $\varepsilon$  attain the coefficient of performance  $\varepsilon_c = T_L / (T_H - T_L)$  of a reversible Carnot refrigerator. However, practical refrigerators always need a certain rate of refrigeration. Therefore, the limits of classical thermodynamics  $\varepsilon_c$  are too rough to predict the coefficient of performance of a practical refrigerator. Under the circumstances of a given rate of refrigeration, the bound of the coefficient of performance of a refrigerator affected by the irreversibility of heat conduction should be determined by using equation (20).

On the other hand, when  $R = K T_L \equiv R_{\max}$ ,  $\varepsilon = 0$ . In such a case, refrigeration cycles cannot be carried out. Consequently, the rate of refrigeration of a practical refrigerator cannot approach  $K T_L$ , and is usually much smaller than it.

These results show clearly that for a practical refrigerator affected by thermal resistance, when  $T_L$  is very low, the maximum coefficient of performance  $\varepsilon_c$  and the maximum rate of refrigeration  $R_{\max}$  are very small. This brings about twofold difficulties in lowering the temperature to ultra-low temperatures. The lower  $T_L$  is, the more serious the difficulty is. Finally, the result will be that the temperature cannot be lowered again. It is thus obvious that the effect of thermal resistance is a serious problem in ultra-low temperature technology. It will be helpful for further understanding of the problem to use the theory of finite-time thermodynamics to make a thorough analysis and study [14].

(iii) We introduce the equivalent temperature

$$T_L^* = T_L - R/K. \quad (21)$$

Then, equation (20) may be written as

$$\varepsilon = T_L^* / (T_H - T_L^*). \quad (22)$$

Also, the rate of minimum average entropy production in a cycle for a given  $R$  may be written as

$$\begin{aligned} \sigma &= \Delta S / \tau = (Q_H / T_H - Q_L / T_L) / \tau \\ &= \frac{Q_L}{\tau} \left( \frac{Q_H}{Q_L T_H} - \frac{1}{T_L} \right) = R \left( \frac{1}{T_L^*} - \frac{1}{T_L} \right). \end{aligned} \quad (23)$$

The form of equation (22) is identical to that of the

coefficient of performance  $\varepsilon_C$  of a reversible Carnot refrigeration cycle, whereas equation (23) corresponds to the rate of entropy production of a heat conduction process between two reservoirs at temperatures  $T_L$  and  $T_L^*$  in which the heat flux is  $R$ . It is thus obvious that under the circumstances of a given  $R$  and  $K$ , the performance of a Carnot refrigeration cycle affected by thermal resistance corresponds to that of a reversible Carnot refrigeration cycle whose low-temperature reservoir temperature is lower than  $T_L$  by  $R/K$ , whereas the irreversibility of heat conduction may be treated as a simple heat conduction process between the reservoirs at temperatures  $T_L$  and  $T_L^*$ . Therefore,  $T_L^*$  is the concentrated expression of the irreversibility of heat conduction in the Carnot refrigeration cycle. When  $T_L$  is small,  $T_L^*$  is much smaller than  $T_L$ . Then, it is seen from equations (22) and (21) that  $\varepsilon$  and  $R$  become very small, and tend to zero as  $T_L$  approaches zero. It is thus clear that  $T_L^*$  also indicates the twofold difficulties in lowering temperature when  $T_L$  is very low.

As mentioned above, owing to the effect of thermal resistance, the coefficient of performance and the rate of refrigeration of a refrigerator must be considered simultaneously. Equation (19) or (20) just provides some theoretical bases for the question of how to reasonably choose the two parameters. For example, when we pay equal attention to both the coefficient of performance  $\varepsilon$  and the rate of refrigeration  $R$ , the multiplication  $\varepsilon R$  may be taken as an objective function. We can then find from equation (19) or (20) that the coefficient of performance and the rate of refrigeration of a refrigerator at maximum  $\varepsilon R$  conditions are respectively given by

$$\varepsilon_0 = [T_H/(T_H - T_L)]^{1/2} - 1 \quad (24)$$

and

$$R_0 = K(T_H - T_L)\varepsilon_0. \quad (25)$$

Thus, we get

$$(\varepsilon R)_{\max} = \varepsilon_0 R_0 = K(T_H - T_L)\varepsilon_0^2. \quad (26)$$

These are the optimal operation conditions of a refrigerator when the coefficient of performance and the rate of refrigeration are paid equal attention.

We can find easily from equations (24) and (25)

$$\varepsilon_0/\varepsilon_C = R_0/R_{\max} = 1/[1 + (1 + \varepsilon_C)^{1/2}] < 1/2. \quad (27)$$

Equation (27) shows clearly that when the coefficient of performance and the rate of refrigeration of a refrigerator are paid equal attention, the coefficient of performance cannot attain the value of  $\varepsilon_C/2$ . Thus, for practical refrigerators, in order to get a certain rate of refrigeration, their coefficients of performance have to be smaller than  $\varepsilon_C$ . However, it is unsuitable if the rate of refrigeration is chosen too large, because the corresponding coefficient of performance would be too small to be beneficial to the reasonable use of energy. In general,  $R$  should be smaller than  $R_0$ . Only in some

special cases, e.g., when a very low temperature is required for the process, can the rate of refrigeration be chosen to be larger than  $R_0$ . Because in such a case,  $R_{\max}$  is very small, heat leakage is quite serious so that a larger rate of refrigeration is needed for the refrigeration. But, this will result in the coefficient of performance becoming reduced, so obviously higher costs must be met. It is thus clear that equation (19) or (20) may provide some theoretical bases which are more significant than the theory of classical thermodynamics for choosing the optimal operation conditions of refrigerators.

(iv) It is interesting to compare the above results with the corresponding results of a Carnot engine. These results are listed in table 1, where  $P$  is power and the subscript m denotes the parameter values at the maximum power conditions. It can be seen clearly from table 1 that the relation between  $\varepsilon_0$  and  $\varepsilon_C$  is similar in form to the relation between  $\eta_m$  and  $\eta_C$ ; the relation between  $(\varepsilon R)_m$  and  $\varepsilon_0$  is similar in form to the relation between  $P_m$  and  $\eta_m$  and so on. Thus it can be seen that there are some common characteristics in using the objective functions  $P$  and  $\varepsilon R$  to discuss the optimal performance of Carnot engines and refrigerators, respectively. Consequently, the parameters  $\varepsilon_0$  and  $R_0$  for practical refrigerators play the same instructive role as the parameters  $\eta_m$  and  $P_m$  for practical engines. They will be helpful to the further understanding of the performance limits of practical apparatus.

#### 4.2. Case $n = 4$

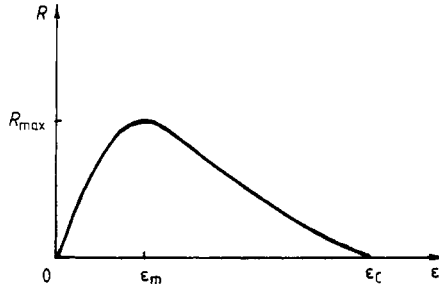
The case  $n = 4$  is a model which is valid for heat transfer by thermal radiation [4]. When  $n = 4$ , equation (17) may be expressed as

$$R = \frac{K_2[T_L^4 - T_H^4/(1 + \varepsilon^{-1})^4]}{\{1 + [K_2/K_1(1 + \varepsilon^{-1})^3]^{1/2}\}^2}. \quad (28)$$

It is shown from equation (28) that  $\partial R/\partial \varepsilon < 0$ . This shows that for the case using the thermal radiation law, the optimal rate of refrigeration  $R$  of a refrigeration cycle is also a monotonically decreasing function of the coefficient of performance  $\varepsilon$ . When  $R = 0$ ,  $\varepsilon = \varepsilon_C$ ; when  $\varepsilon = 0$ ,  $R = K_2 T_L^4 \equiv R_{\max}$ . This also shows that only if the rate of refrigeration is equal to zero can the coefficient of performance attain the value  $\varepsilon_C$  of reversible thermodynamics for a Carnot refrigerator affected by thermal resistance. In fact, this is a general conclusion which is independent of the heat transfer law. Its physical meaning is obvious, because it will always be restricted by the second law of thermodynamics whichever heat transfer law is used. According to the second law of thermodynamics, heat can only flow spontaneously from a hotter to a cooler body. Consequently, only if the relation  $T_1 > T_H > T_L > T_2$  is satisfied can the heat conduction be carried out in finite time. On the other hand, the value of  $R_{\max}$  is obviously dependent on the heat transfer law. As shown above,  $R_{\max}$  is independent of  $K_1$  in the case of

**Table 1.** The corresponding relations between the Carnot engine and the refrigerator.

	Carnot engine	Carnot refrigerator
Reversible	$\eta_C = 1 - T_L/T_H$ $Q_H/\tau = 0$ $P = 0$	$\varepsilon_C = T_H/(T_H - T_L) - 1$ $R = Q_L/\tau = 0$ $\varepsilon R = 0$
Irreversible	$P_{\max} = KT_H\eta_m^2$ $\eta_m = 1 - (T_L/T_H)^{1/2}$ $(Q_H/\tau)_m = KT_H\eta_m$	$(\varepsilon R)_{\max} = K(T_H - T_L)\varepsilon_0^2$ $\varepsilon_0 = [T_H/(T_H - T_L)]^{1/2} - 1$ $R_0 = K(T_H - T_L)\varepsilon_0$

**Figure 3.** The curve  $R$  varying with  $\varepsilon$  for  $n = -1$ .

$n = 4$ , but is dependent on  $K_1$  for the case  $n = 1$ . For some heat transfer laws, even the optimal rate of refrigeration  $R$  is not a monotonic function of  $\varepsilon$ . For example, when  $n = -1$ , the relation between  $R$  and  $\varepsilon$  is just the circumstances (see figure 3).

#### 4.3. Case $n = -1$

The case  $n = -1$  is another linear model in irreversible thermodynamics which is also used in practice and finite time thermodynamics [1–3, 5]. When  $n = -1$ , equation (17) may be expressed as

$$R = -K_2 \frac{\varepsilon(1 + \varepsilon)}{[\varepsilon + (K_2/K_1)^{1/2}(1 + \varepsilon)]^2 T_H T_L} \times \left( T_L - \frac{\varepsilon}{1 + \varepsilon} T_H \right) \quad (29)$$

where  $-K_1$  and  $-K_2$  are the so-called kinetic coefficients [15]. It is seen from equation (29) that  $\varepsilon = \varepsilon_C$  when  $R = 0$ . This shows again that it is a general conclusion.

On the other hand,  $R$  expressed by equation (29) is not a monotonic function of  $\varepsilon$ , and there exists an extreme value  $R_{\max}$ , as shown in figure 3. From the extremal condition  $\partial R/\partial \varepsilon = 0$ , we obtain that when

$$\varepsilon = T_L[2T_H + (K_1/K_2)^{1/2}T_L - T_L] \equiv \varepsilon_m \quad (30)$$

$R$  attains the maximum, i.e.

$$R_{\max} = -K_1 T_L/[4T_H^2 + 4(K_1/K_2)^{1/2} T_H T_L]. \quad (31)$$

When  $R < R_{\max}$ , under a given  $R$ , we can get two values of  $\varepsilon$  from equation (29), where one is larger than  $\varepsilon_m$  and the other is smaller than  $\varepsilon_m$ . Evidently, only the larger one is the optimal coefficient of performance under the given  $R$ . Therefore, the optimal coefficient

of performance should lie between  $\varepsilon_m$  and  $\varepsilon_C$ . The important significance of  $\varepsilon_m$  lies in the fact that it determines a lower limit of the value of the optimal coefficient of performance for Carnot refrigeration cycles in this case.

In addition, it can be seen from equation (30) that  $\varepsilon_m$  is not only a function of reservoir temperatures but is also dependent on the ratio of heat conductances  $K_1/K_2$ . This matter is worth noting in the study of cycles affected by thermal resistance, otherwise the conclusions would lose their universality [5]. The particular importance of the identification of heat transfer laws in real thermodynamic devices can be inferred from the possibility of the existence of different heat transfer laws.

#### 5. Relation between $\varepsilon$ and other parameters

(i) Owing to the relation between the average power input  $P$  and the rate of refrigeration  $R$  of a refrigeration cycle being  $P = R/\varepsilon$ , we get

$$P = K_2 \frac{T_L^n - T_H^n/(1 + \varepsilon^{-1})^n}{\varepsilon\{1 + [K_2/K_1(1 + \varepsilon^{-1})^{n-1}]^{1/2}\}^2} \quad (32)$$

from equation (17). Equation (32) determines the relation between the optimal coefficient of performance and the power input of a Carnot refrigeration cycle with the only irreversibility of heat conduction.

(ii) Owing to the rate of average entropy production in a cycle being

$$\begin{aligned} \sigma &= \Delta S/\tau = (Q_1/T_H - Q_2/T_L)/\tau \\ &= R[(1 + \varepsilon^{-1})/T_H - 1/T_L] \end{aligned} \quad (33)$$

we get from equation (17) that the relation between the optimal coefficient of performance and the rate of average entropy is

$$\sigma = K_2 \frac{T_L^n - T_H^n/(1 + \varepsilon^{-1})^n}{\{1 + [K_2/K_1(1 + \varepsilon^{-1})^{n-1}]^{1/2}\}^2} \left( \frac{1 + \varepsilon}{\varepsilon T_H} - \frac{1}{T_L} \right). \quad (34)$$

Equation (34) indicates that no matter how the value of  $n$  is chosen, i.e., no matter which heat transfer law is used, we can have no irreversible loss (i.e.,  $\sigma = 0$ ) if, and only if,  $\varepsilon = \varepsilon_C$ . Therefore the irreversible loss is inevitable in a practical refrigerator.

The relations between  $\varepsilon$  and other parameters besides those mentioned above, e.g., the relation between the optimal coefficient of performance and the rate of loss of availability, etc, are also determined from equation (17). In other words, equation (17) can play a more instructive role than the theory of classical thermodynamics in the optimal design of refrigerators.

## 6. Conclusion

It is shown from the above discussion that even though the performances of Carnot refrigeration cycles are different from each other for different heat transfer laws (for different value of  $n$ ), the optimal coefficient of performance always decreases as the rate of refrigeration increases, it can attain the bound  $\varepsilon_C$  of classical thermodynamics if, and only if, the rate of refrigeration is equal to zero, and the irreversible loss of the finite rate of heat conduction is inevitable. These all result from the second law of thermodynamics. Different heat transfer laws can only change the value of the lower limit of the optimal coefficient of performance, the maximum rate of refrigeration  $R_{\max}$  and other relative quantities. Moreover, for a concrete heat transfer law, in general, these quantities are also dependent on the

heat conductances. Therefore, for a practical refrigerator, the suitable heat transfer law, the reasonable coefficient of performance and rate of refrigeration should be chosen in accordance with the particular circumstances such that the refrigerator can operate in the optimum conditions. The conclusions of finite-time thermodynamics can just provide some new theoretical bases for such an optimal design.

## References

- [1] Orlov V N 1985 *Sov. Phys.-Dokl.* **30** 506
- [2] Vos A D 1985 *Am. J. Phys.* **53** 570
- [3] Orlov V N and Rudenko A V 1985 *Autom. Remote Control* **46** 549
- [4] Vos A D 1987 *J. Phys. D: Appl. Phys.* **20** 232
- [5] Chen L and Yan Z 1989 *J. Chem. Phys.* **90** 3740
- [6] Curzon F L and Ahlborn B 1975 *Am. J. Phys.* **43** 22
- [7] Rubin M H and Andresen B 1982 *J. Appl. Phys.* **53** 1
- [8] Rubin M H 1979 *Phys. Rev. A* **19** 1272
- [9] Salamon P and Nitzan A 1981 *J. Chem. Phys.* **74** 3546
- [10] Rebhan E and Ahlborn B 1987 *Am. J. Phys.* **55** 423
- [11] Ondrechen M J, Rubin M H and Band Y B 1983 *J. Chem. Phys.* **78** 4721
- [12] Chen J and Yan Z 1988 *J. Appl. Phys.* **63** 4795
- [13] Yan Z and Chen J 1989 *J. Appl. Phys.* **65** 1
- [14] Yan Z 1984 *Cryogenics* (in Chinese) **3** 17
- [15] Callen H 1960 *Thermodynamics* (New York: Wiley)